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Analysis of Bus Vulnerability Conducted Using a Synchronized Phasor Measurement Unit in Order to Achieve the Maximum Observability

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Abstract: Phasor measurement units (PMUs) have gained significant interest in recent decades. These instruments are used to measure synchronized phasor data. PMUs are gradually but definitely taking over power grids because of the significant phasor information that they generate for both regular and irregular conditions for the purpose of maintaining safety and control. PMUs may be used for a variety of purposes, including state estimation, which is a common task. In order to make state estimation more reliable, a variety of approaches have been looked into, and one of them is the positioning of PMUs. This paper provides a plan for the implementation of the PMUs, taking into account the potential for failure and vulnerability posed by PMU-equipped buses. Two separate studies were carried out and evaluated with the goal of solving the optimum PMU placement problem (OPPP), which pertains to the grids. The findings of the first study show that the maximum bus observability may be accomplished with the fewest possible number of PMUs, even while taking into consideration the fact that there is a risk that one or more PMUs would malfunction. This investigation was carried out with common measures such as zero injection bus (ZIB) and branch flow measurements, both with and without them, in order to assess the outcomes. The second research focused on selecting the PMU-equipped bus's vulnerability analysis as its primary area of investigation. All of the tests were completed by using binary integer linear programming. Specifically, the described method is meant to be used with an existing PMU framework and in the case that new locations for new PMUs are necessary to be furnished with existing PMUs. This results confirm that the recommended strategy can be implemented successfully on the IEEE benchmark test systems.

1. Introduction

Present-day power grids are increasingly complicated and constrained owing to the demands of intensive energy usage [1]. The majority of the high-cost blackouts are a direct consequence of these demanding

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circumstances. In order to effectively manage the power grid's resources, a real-time wide-area monitoring, security, and control system (WAMS) is required. WAMS provides system security and smooth operation for operators. State estimation is essential for the energy management system [2]. State estimation offers a valuation of the power grid's state variables based on measurements acquired across the network, thereby ensuring that these predictions are compatible with the sizes. SCADA (Supervisory Control and Data Acquisition) has traditionally provided input measurements. One of its limitation is that the measurements are uncoordinated, leading to unreliable state estimation during dynamic network phenomena. Synchronized phasor measurements are possible with the advent of phasor measurement units (PMU), which allow for dynamic phenomena monitoring. Amongst the different applications of PMUs, the state estimator is one of the most impacted variables. PMUs are supremely effective for WAMS performance in order to improve state estimation [3]. To ensure that measurements of positive sequence electrical quantities are synchronized to a standard time frame, the PMU makes use of global positioning system (GPS) technology [4].

In order to find the ideal values for the bus voltage phasors, the state estimator must take into account the results of the measurements as well as the network architecture of the electric power grid. On-site measurements are made using remote terminal units at the substation. These measurements include active and reactive power flows, power injections, bus voltage magnitudes, and branch currents [5]. Currently, industrial PMUs are readily available in the marketplace, some companies or energy businesses have started installing PMUs in the power grid, and more will soon be established [6]. But, the prohibitive cost of PMUs is a taxing concern. PMUs have evolved into an indispensable device for ensuring the uninterrupted operation of the power grid, enhancing the efficiency of power flow and facilitating the incorporation of renewable energy sources. Nevertheless, there are expenses associated with installing PMUs, and these costs can vary depending on a number of factors, including the cost of the hardware, installation and commissioning, data connection, maintenance and calibration, integration, and security.

Regarding the practicability of managing PMU costs in real-life large-scale power networks, it is essential to keep in mind that the advantages of PMU deployment frequently surpass the costs. PMUs deliver critical data in real-time, which grid operators may utilize to identify and respond to disruptions, avoid blackouts, and make the most use of grid assets. They are also very helpful in the process of upgrading power systems and strengthening their resistance to disruption.

However, the viability of widespread deployment of PMUs is contingent on several circumstances, including the financial resources of the utility or grid operator, the backing of regulatory authorities, and the unique requirements of the power network. The deployment of PMUs is an investment seen as cost-effective by many sophisticated power systems. This is especially true for systems that have a large percentage of renewable energy sources and complicated grid arrangements. The number of PMUs installed has a significant impact on the observability of the electric grid [7].

An unexpected outage occurring in a network or in a PMU has an impact on the observability of the system and has the potential to cause a substantial issue. Due to the obvious pertinent structure of electrical networks, all buses must be extensively and continuously monitored. The optimal PMU placement problem (OPPP) is a key topic of research that is currently being pursued. This research work is restricted to the OPPP only. Researchers and power engineers have used stochastic and deterministic approaches to the OPPP worldwide.

In [8] and [9], the formulation for evaluating the OPPP is based on integer linear programming (ILP), and linear constraints are constructed based on the vertex-to-vertex connection matrix. It was the first formulation to be developed, and it simplifies the assessment of power grid observability when taking into account zero injection buses (ZIBs). However, the previously stated constraints become nonlinear [10]. Bei Gou suggested an ILP method in [11] and [12] that considered both traditional calculation and ZIB, similar to the work presented in [9]. The proposed solution removes nonlinear constraints using a permutation matrix. In [6], the authors suggested an automated method to unravel PMU's minimal number and strategic positions to allow the power grid to be wholly controlled and thus be used for state estimation. Binary integer linear programming (BILP) is used to solve the OPPP problem, in which discrete variables are used to achieve an optimal location for a PMU. In [13], Pal et al. proposed that a dual approach could be used to unravel the answer to the OPPP in order to attain maximal observability across a vast network. When it came to dividing the tree into sub-trees, the algorithmic procedure employed a binary search strategy. By using this strategy, the ILP spanning tree may be split apart. The ILP was calculated using information about power grid connections from vertex-to-vertex. After disintegration, PMU was strategically placed in subnetworks to reduce PMU placement costs. In [14], the OPPP methodology hinges on the topological observability approach furnished by graph theory formulation. The depth-first search (DFS) technique is used to assess the OPPP responses [15, 16]. However, DFS has the downside of not exploring the node, only detecting the node in the tree. Moreover, it provides only one optimal solution. This limitation was addressed by developing a two-stage branch-and-bound algorithm (BBA) to circumvent it in [17]. In stage 1, BBA used DFS to visit the specific node, and in stage 2, BBA used best-first search (BFS) to explore the node and detect the anticipated target.

In [18] and [19], the use of a mixed integer linear program (MILP) is advised for developing effective solutions for the OPPP when taking into account the findings of zero injection measurements, which include line and PMU failures. In [20], Aghaei et al. developed a mixed-integer programming (IP) probabilistic approach for the OPPP to ensure full monitoring of the power grid. This offers a practical linearization approach to transform nonlinear expressions into linear expressions to unravel the problem. Mahari and Seyedi [21] developed a novel OPPP approach using ZIB to ensure power grids' observability. The binary-ILP-based approach is proposed in [20] for the OPPP to attain complete grid monitoring along with optimized redundancy measurement. Chatterjee et al. [22] reveal a three-fold OPPP based approach. Initially, PMUs are positioned on every linked network bus. Later, redundant bus positions are selected, and the most promising buses where tree-based pruning applications retain PMUs are concentrated upon. In [23], the authors developed a state estimation OPPP technique based on bus-to-bus connection data, which takes into account a single PMU and a line interruption, among other considerations. The solution, on the other hand, does not seem to be encouraging. In [24] and [25], an admissible search-based approach, such as the A-star or binary search tree, is recommended for unravelling the OPPP in order to achieve total connected power network observability that considers all possible scenarios and eventualities. The OPPP is intended to limit the number of PMUs that may be installed on the power grid while still guaranteeing that the grid's overall observability is maintained [26]. The OPPP is unraveled using a modified BBA [27].

In [28], the authors present an integer quadratic programming approach for determining the most advantageous PMU sites while improving the redundancy measurement. The paper [29] proposes an ILP-based technique for power grid observability, which takes into account OPPP and a fixed PMU channel of communication. Sodhi et al. [30] proposed an IP for classifying the OPPP for maximal power grid observability and investigates a multi-criteria decision-making approach for identifying the most viable areas for deployment, minimizing the appropriate PMU numbers and optimization of redundancy measurement at each bus is achieved. Compared to the ILP approaches, stochastic approaches depend on fine-tuning parameters to ensure the most promising solutions. Furthermore, depending on the number of goals, each goal may be shaped as a part of a fitness function. Each solution obtained by the algorithms is evaluated to ensure its correlation to the desired destination. However, the computation time may increase depending on the size of the network. It is possible that it will not reach an optimum solution for a wide network issue.

In [31], a binary coded genetic algorithm (BCGA) was proposed for OPPP based on two-fold objectives, minimizing the number of PMUs while optimizing redundancy measurement. Initially, BCGA used the graph-theoretical method and standard GA to find optimal solutions. BCGA was then utilized to estimate the non-dominant solution giving the best trade-off between two-fold targets. Ref. [32] proposes the topology-based approach that uses a genetic algorithm (GA) to unravel the OPPP. This research work includes ZIB integration with one of its neighboring buses. The results indicate that this approach may be used to address the OPPP. According to the findings of the research study by Zhou et al. [33], a GA immunity technique may be used to determine the minimal number and strategic placements of PMUs necessary to adequately monitor the power grid. The results show it converges faster than the regular GA. Considering the accessibility of channel capacity, along with the possibility of a single line outage, a novel cellular GA-based technique for OPPP is studied in [34]. Since nature-inspired GA can provide multiple solutions, only one set of solutions is considered. The differential evolution-based method was proposed in [35] to comprehend the PMU injection bus and redundancy measurement concerns. The propounded process was simulated on IEEE systems and was compared to other available research. In [36], Peng et al. posed a novel Pareto-based non-dominated sorting technique for assessing a multi-objective OPP problem considering a PMU outage. They affirmed that many Pareto-optimal solutions can be found using this methodology and that its versatility allows consideration of further goals for future study.

Work in [37, 38] suggested the binary particle swarm optimization (BPSO) to unravel OPPP with and without ZIBs. Redundancy measurement was proposed to rate solutions. An improved-BPSO (IBPSO) for the power grid as proposed in [39] and [40] was adopted for the OPPP. IBPSO provides additional speed upgrade rules in case the particles cannot offer any promising solution. This method's essence is to ensure that the knowledge of feasible solutions can always be used to guide the swarm to decide the best solution. If none of the particles can come up with a feasible solution, a random search is used to increase the likelihood of finding a quick and optimal solution. Ref. [41], puts forward a novel approach for regulating inertia weight for BPSO. Compared to conventional linear inertia weight, inertia weight would decrease exponentially by adopting the process. The authors believed that it advances the algorithm search capability. Also, the proposed approach included a mutation operator.

In order to identify the OPPP, researchers and engineers have proposed various approaches and strategies accounting for contingencies [42, 43]. Yet, most of them deal only with OPPP but neglect the possibility of failure of PMU equipped buses. While PMUs are very accurate in providing reliable data, they might not be accessible owing to communication failures. Real time situations make PMUs prone in this regard. Therefore, our objective is to propose a method for OPPP considering the bus vulnerability analysis of PMU equipped buses and its possible failure for maximum observability of the power grid. The following are the contributions made by this analysis:

1. In the absence of traditional measurements, a fixed number of PMUs are installed considering the possible failure of the PMU equipped buses and its vulnerability analysis.
2. In the presence of traditional measurements such as zero injection and branch flow measurements, a set number of PMUs shall be installed considering the failure of the PMU equipped buses.

There are five sections to this article. Section 1 offers context on PMUs and their significance in the state estimation. In addition, a literature survey of OPPP methodologies and goals is also provided. An in-depth explanation of the theory that was used in the OPPP technique, both with and without traditional measurements, is given in Section 2. Section 3 describes the position of the PMU considering the possibility of its failure. The possible impact of taking traditional measurements into account is also discussed. Case studies and the findings derived from the method are discussed in Section 4. Section 5 concludes the work by summarizing it.

2. Mathematical formulation of the ILP for OPPP

In recent times, ILP has gained prominence in many practical applications, including planning, scheduling, telecommunications, and more, probably owing to tremendous improvement in rapid computation and updated algorithms. According to experts, ILP serves as the foundation for a variety of analytical decision-making processes [44]. A primary body consists of three main parts: variables, restrictions, and the goal function itself. Variables are used to make decisions about integer values, whereas constraints are used to keep values within a reasonable range. Constraints should always be linear when constructing an ILP formula. Either linear equality or linear inequality, or both, are possible. Depending on the circumstances, the objective function specifies whether the ideal solution should be maximized or minimized by the objective function. In addition, the objective function must be linear in nature.

Utilizing ILP to search OPPP is trending since it saves CPU processing time [5]. In this paper, the OPPP issue is solved using the BILP method. Between ILP and BILP, BILP's decision variables are binary values [0, 1] . The general formulation is categorized as follows: (i) Absence of conventional measurement and (ii) Presence of conventional measurement.

2.1. Absence of conventional measurement

In this section, branch flow and injection measurements are not taken into account. The methodology for OPPP for complete observability is formulated as

$$\min \sum_{i=1}^{n} W \cdot x_i \ , \tag{1}$$

subject to

$$f(X) = A \cdot X \geq b \ , \tag{2}$$

$$W = [1 \quad 1 \ldots 1]_{1\times n} \ , \tag{3}$$

$$X = [x_1 \quad x_2 \ldots x_n]^T \ , \tag{4}$$

and

$$x_i \in \{0, 1\} \ , \tag{5}$$

where x_i is the vector of binary decision-making variables, which is given as

$$x_i = \begin{cases} 1 & \text{if PMU is installed at bus } i \\ 0 & \text{Otherwise,} \end{cases} \tag{6}$$

W is the weight or cost function; n is the total number of buses in the power grid; A is the bus-to-bus connection matrix information of power grid and is obtained as

$$A_{ij} = \begin{cases} 1 & \text{if } i \text{ and } j \text{ are adjacent} \\ 1 & \text{if } i = j \\ 0 & \text{Otherwise,} \end{cases} \tag{7}$$

and $b(N \times 1)$ is the column vector, and all the entries in a column vector are ones.

The objective function, represented by Equation 1, accounts for the minimal number of PMUs required to achieve total power grid observability while maintaining accuracy. In this case, the cost coefficient for PMUs is considered to be unity, which means that the cost for each independent PMU is postulated to be the same [45]. In Equation 2, inequality constraints describe that respective buses in the power grid must be observed at least once by PMU equipped bus. The A matrix represents the binary connection matrix from bus-to-bus. This constraint guarantees that the electrical grid will be observed in its entirety. The number of essential constraints is $n \times n$. The solution of this OPPP has the smallest number of PMUs and their related optimum placements in the solution space. OPPP is shown using the 7-bus power grid as an example, which is discussed further below. Figure 1 shows a simplified single-line schematic of a 7-bus network.

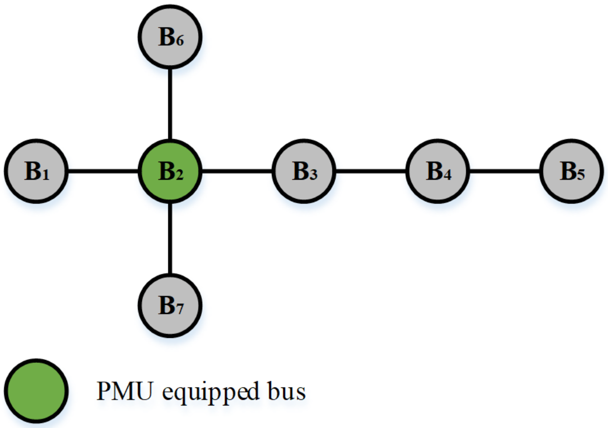

Figure 1. Single line diagram of 7- bus network.

In the case of a 7-bus system, the bus-to-bus binary connection matrix may indeed be derived by solving Equation 6. Now multiplying

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{8}$$

with the decision variables, we get

$$f(X) = A \cdot X = \begin{cases} f_1 = x + x & \geq 1 \\ f_2 = x_1 + x + x_3 + x_6 + x_7 & \geq 1 \\ f_3 = x_2 + x + x_4 & \geq 1 \\ f_4 = x_3 + x_4 + x_5 & \geq 1 \\ f_5 = x + x_5 & \geq 1 \\ f_6 = x_2 + x_6 & \geq 1 \\ f_7 = x_2 + x_7 & \geq 1. \end{cases} \tag{9}$$

The $+$ operator sign is the logical OR, and the observability constraint is the right-hand side of all the inequality constraints. This implies that the variables must be 1 or higher in each row, which includes a summation. For example, the very first row shows bus 1. PMUs can be put on bus 1 or bus 2 according to constraints to guarantee that bus 1 can be observed. Likewise, the PMU might be positioned on bus 1, 2, 3, 6, or 7 in order to make bus 2 and subsequent buses observable. For this method, the corresponding optimum PMU count is two and the better places are bus 2 and bus 4. The positioning then allows the buses to complete observability.

2.2. Presence of conventional measurement

Conventional measurements are considered an inexpensive approach to reduce the number of PMUs in the power grid. As articulated, conventional measurements in this article refer to measurements such as zero injection and measurements of branch flow. Branch flow measurements between two buses are already available in realistic power grids, and installation costs are significantly lower than PMUs for branch measurements. Also, bus voltage angle can be determined by utilizing the branch flow measurement. In ZIBs, zero injection measurements are found. Any bus that does not have a load or generator is regarded as ZIBs. These ZIBs require no measurement and are considered to be accurate state estimations [5]. They are also considered pseudo-measurement. There are some rules for grid observability, considering ZIBs [46].

1. If a ZIB can be observed and the neighboring buses can all be observed except for one bus, a non-observable bus can be observed by applying the KCL.
2. If all buses except for the ZIB are observed, then a certain ZIB can be observed by employing nodal equations.

2.2.1. ZIB model

In order to illustrate the conventional measurement based on power grid observability, a 6-bus power grid, as displayed in Figure 2, is taken into consideration. To validate the model, assume a vector

$$H = A \cdot X \ , \tag{10}$$

where H designates the redundancy measurement in which the element is $h_{i,j} = A_{i,j} \cdot x_i$, while X is the matrix for the positioning of PMUs in columns in which x_i is the i-th component, A is the binary connection matrix for bus-to-bus network in which $A_{i,j}$ is the i-th row and j-th column. Redundancy measurement H of a bus means that installed PMUs observe the number of times the bus is observed. The redundancy measurement of the system can also be calculated by adding up individual bus measurement redundancy.

Due to the presence of conventional measurement, the resulting observability criteria must be analyzed with three criteria.

Case i. Flow measurement

As shown in Figure 2, this measurement may be used to observe both bus 2 and bus 3, whereas the PMU must be used to monitor the other buses. This can be expressed as

$$h_2 + h_3 \geq 1 \ . \tag{11}$$

Case ii. Zero-injection measurement

In the context of the zero-injection measurement laws stated above, the PMU should observe a minimum of four buses out of five buses, as given by

$$h_1 + h_2 + h_3 + h_5 + h_6 \geq 4 \ . \tag{12}$$

Case iii. Hybrid measurement

The branch flow measurement and zero-injection measurement are combined in this research to form a hybrid measurement method (bus 2, as presented in Figure 2). All buses except one may be observed by the PMU, excluding those used for conventional measurement. Observability may be achieved in some

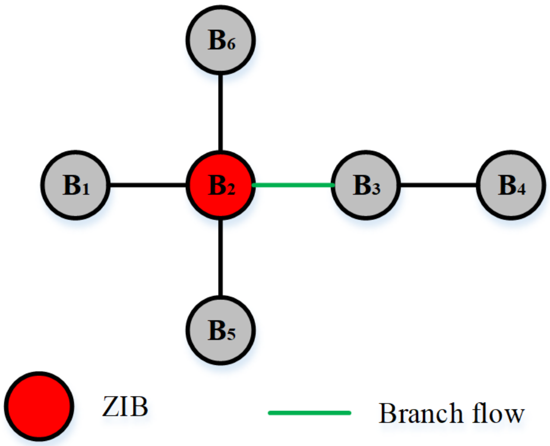

Figure 2. A single line schematic of a 6-bus network.

situations by using a hybrid measurement method, which is available on one bus. This can be represented as

$$h_1 + h_5 + h_6 \geq 2 \ . \tag{13}$$

The right side of Equations 11, 12 and 13 show the number of buses the PMU can observe.

2.2.2. Mathematical formulation considering conventional measurements

It is evident from Section 2.2.1 of the observability model that the device with buses that are involved in conventional measurements may be split into two categories: buses that are correlated with conventional measurements and buses that are not correlated with conventional measurements [47]. Consequently, while defining the constraint for OPPP, the categorized buses should be evaluated in sequence, with the bus that is not linked with traditional measurement being considered first in the order of consideration. The following mathematical formulation pertains to conventional measurements:

$$\min \sum_{i=1}^{n} W \cdot x_i \ , \tag{14}$$

subject to

$$\begin{bmatrix} I_{M\times M} & 0 \\ 0 & T_{meas} \end{bmatrix} (PH) = T_{con} P(A \cdot X) \geq b_{con} \ , \tag{15}$$

where T_{meas} and b_{con} are aggregate of buses related to traditional measurements, these are deduced as stated in the three cases mentioned above; M is the amount of buses that are not related according to the traditional measurements and P is a matrix of permutations. For illustrative purpose, the 7- bus network exhibited in Figure 3 is considered as an example.

In accordance to the formulation of the bus-to-bus binary connectivity matrix, as given in Equation 8, these two conventional measurements are not related with buses 4 and 5. Constraints on equality that relate to the two conventional measurements are as follows:

$$h_2 + h_3 \geq 1, \tag{16}$$

and

$$h_1 + h_6 + h_7 \geq 2 \ . \tag{17}$$

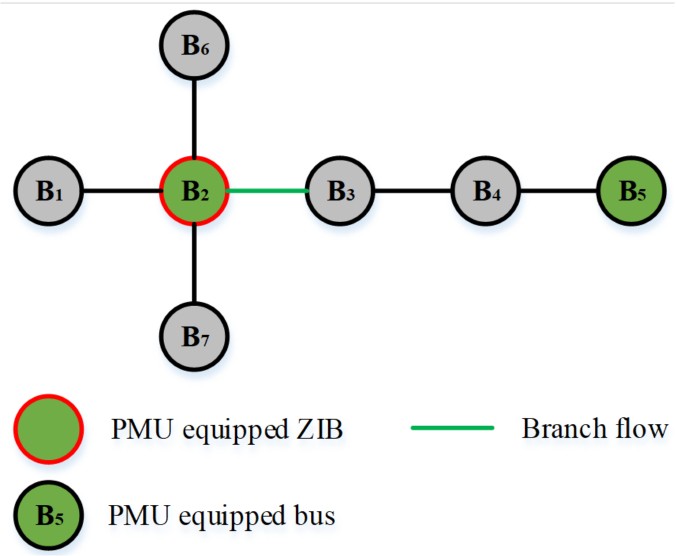

Figure 3. A single line schematic of a 7-bus network with conventional measurements.

By means of the previous inequalities, T_{meas} can be given as

$$T_{meas} = \begin{array}{c} \begin{array}{ccccc} 1 & 2 & 3 & 6 & 7 \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 \end{bmatrix} \end{array} \quad \begin{array}{l} \text{Buses} \\ \text{Branch meas. 2-3} \\ \text{Injection meas. 2} \end{array} \quad . \tag{18}$$

$I_{M \times M}$ is the unity matrix, where

$$M = (\text{number of the buses in the power grid}) - (\text{number of column in } T_{meas}) \ . \tag{19}$$

In this case, $I_{M \times M}$ would be $I_{2 \times 2}$ and thus, T_{con} is expressed as

$$T_{con} = \left[\begin{array}{cc|ccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ \hline 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 1 \end{array} \right] \ . \tag{20}$$

In this case, buses that are not related to measurements are buses 4 and 5. Therefore, the permutation matrix would be written as

$$P = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix} \ . \tag{21}$$

On performing the matrix multiplication of T_{con}, P, A and X given by Equations 20, 21 and 8, respectively, the BILP formulation can be expressed as

$$T_{con} \times P \times A \times X = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 & 1 \\ 1 & 2 & 2 & 0 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 & 3 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix} \geq \begin{bmatrix} 1 \\ 1 \\ 1 \\ 2 \end{bmatrix} \ . \tag{22}$$

The resultant optimum number of PMUs is two and PMU is strategically located on bus 2 and bus 5. The usage of conventional measurements in the installation of PMUs may be seen in this example. Because a minimally acceptable method was used, the optimal number of PMUs with and without traditional measurements was the same in both cases. However, the PMU position is different.

3. OPPP for maximum observability considering its possible failure

It is the goal of this research to increase bus observability by resolving the PMU failure using the OPPP technique. This is intended by evaluating PMU loss and its effect on the observability of power grids. Enforcing this approach necessitates more PMUs compared to the standard operating conditions. Since PMUs are costly, the solution is equally versatile for each test system to increase the PMU necessity to only one more than required; for a standard functioning environment. The possibility of failure of each PMU installed in the network is considered. Each PMU's failure is assessed regularly so that only one PMU malfunction is measured every time. This is due to the fact that a single PMU outage happens more often than two PMU outages. In this research work, the model of the optimization problem is separated into two cases, i.e., with and without conventional measurement.

Case 1: Without conventional measurement

The OPPP suggested here is used to examine PMU failure. This problem includes two parts. The first is the development of n_{PMU}, which refers to the optimum number of PMUs necessary to achieve complete observability in the power grid. Then, taking into account the malfunction of $(n_{PMU} + 1)$ one by one and addressing the issues in Equations 23 to 26. The mathematical formulation is given as

$$\max(\sum_{i=1}^{n} x_i + \sum_{i=n+1}^{n_{nze}} x'_i) \ , \tag{23}$$

subject to

$$f(X) = A \cdot X \geq b \ , \tag{24}$$

$$x'_i \times T_{\min,i} \leq b'_i \leq x'_i \times T_{\max,i} \ , \tag{25}$$

and

$$X = \begin{bmatrix} x_1 & x_2 & x_i & x_n \end{bmatrix}^T \ . \tag{26}$$

Furthermore, the entries of x are given by

$$x'_i = \begin{cases} 1 & \text{if } b'_i = 0 \\ 0 & \text{if } b^i_i > 0 \end{cases} \ , \tag{27}$$

where x_i is the decision variable for PMU installation, x'_i is a binary decision variable that represents the only buses observed by PMUs; n_{nze} is the number of nonzero elements of bus-to-bus binary connectivity matrix A; while $T_{min,i}$ is the minimal number of nonzero components of the i-th bus, which is preferred as 1; $T_{max,i}$ is the total number of nonzero components of bus-to-bus binary connectivity matrix A corresponding to the i-th bus; and b_i' is the A and X result after eliminating one of the PMUs.

Equation 23 provides the total number of PMUs that are available and the highest number of non-zero components of matrix A actually observed by PMUs. As a result of the equality constraint in Equation 24 at least one PMU must be assigned to each bus for observation. The equality constraint in Equation 25 is the number of PMUs fitted in the power grid.

Case 2: With conventional measurement

The OPPP aims to evaluate the potential failure of PMUs by installing a restricted number of PMUs corresponding to their costs. For simplicity, it is assumed that the cost is unity for every PMU and that all of them have adequate channel capacity to monitor the current phasors of adjacent buses. The optimization model considering conventional measurements can be given as

$$\max\left(\sum_{i=1}^{n} w_i x_i + \sum_{i=n+1}^{n_{nze}} x_i^i\right) \ , \tag{28}$$

subject to

$$T_{con} P(A \cdot X) \geq b_{con} \ , \tag{29}$$

$$x_i^i \times b_{con,i} \leq b_i' \leq x_i' \times T_{\max,i} \ , \tag{30}$$

and

$$X = \begin{bmatrix} x_1 & x_2 & x_i & x_n \end{bmatrix}^T \ , \tag{31}$$

where w is the cost function explained by a row matrix of ones: $[1\ 1\ 1\ 1\ ...]_{1\times n}$. All other vector and variable matrixes are as shown in the previous section. The only distinction between the suggested positioning strategies considering traditional and non-conventional calculation is a limitation of Equations 29 and 30. Section 2 addressed the information interpretation for Equation 29. Regarding the mathematical description in case 1 of this segment, when considering devices that don't rely on traditional calculations, observability restrictions vary among buses and are not uniform. Because of zero injection and flow estimation, it is important that at least one PMU can observe each bus. In some cases, the maximum observability value for specific buses might be 0, 1, or 2, depending on whether or not they are coupled with zero injection and flow estimation. Figure 4 depicts a flowchart of the algorithmic process for the suggested approach, which explains how it works.

The algorithmic process to solve the OPPP is illustrated in the following steps:

1. Read power grid bus data and line data.
2. Determine the bus-to-bus connectivity matrix A.
3. Optimize the PMU position for constraints in Equations 24 and 25.
4. Delete one of the system's installed PMUs to evaluate possible failure when OPPP solutions are obtained. This algorithmic perspective is done by deleting the i-th column from the bus-to-bus binary connectivity matrix, similar to removing i-th bus PMU.
5. Following optimization, the number of observed nodes served by PMUs is obtained.
6. If full observability is achieved considering bus observability index (BOI) and system observability redundancy index (SORI), the process stops. If this is the case, refresh the PMU position again and proceed through steps 4 to 5 before the program concludes with the best possible result.

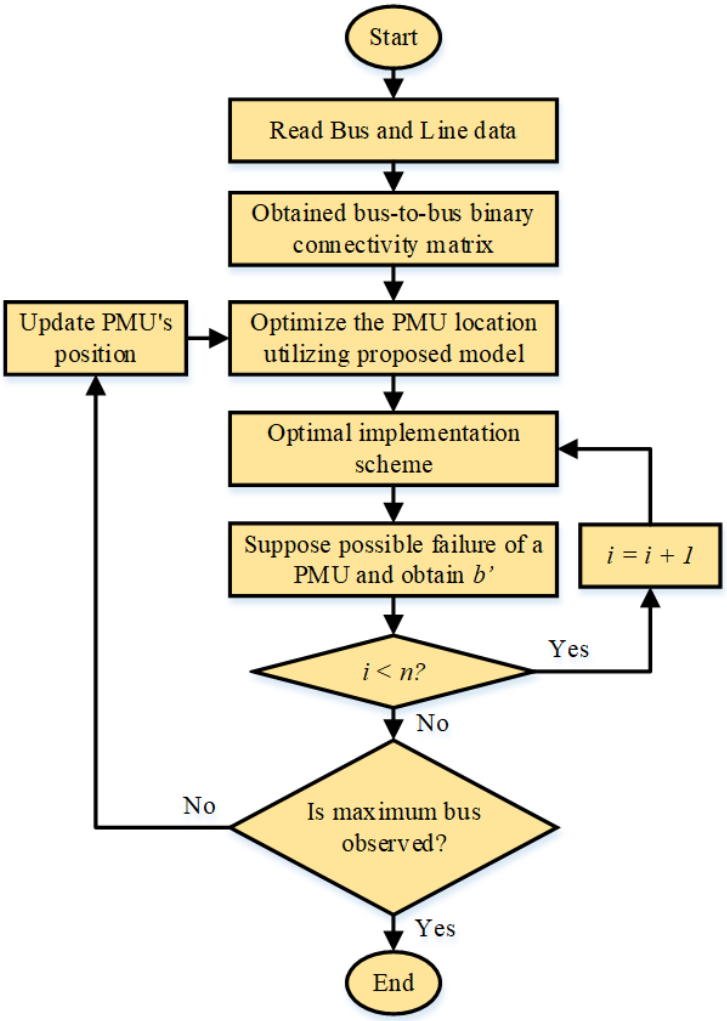

Figure 4. Flowchart illustrating the suggested method's algorithmic view.

When examining the outcome of removing a PMU from a bus i, it is important to remember that, instead of looking at all of the buses, just the ones that are attached are evaluated. The number of variables and constraint equations has been significantly decreased. This is specially evident when the large scale network is involved.

4. Results and discussion

The optimization challenge discussed in the preceding sections was developed and implemented in MATLAB software on a 2.0 GHz clock-speed processor Intel Core i3 device, which was backed by 8 GB of RAM. The proposed algorithm for the OPP problem was implemented on standard IEEE 14-bus, IEEE 57-bus, IEEE 118- bus test networks. An overview of the test networks can be represented as a single-line figure from [48] and [49]. The optimization is carried out within the Matlab framework, utilising the bintprog toolbox, which is based on binary integer linear programming [50].

Table 1 describes the configuration for the standard test networks. There are 20 branches in the IEEE 14-bus system, as well as 1 zero injection bus (ZIB) and one radial buses (RBs). A total of 80 branches and

15 ZIBs are available on the IEEE 57-bus system. Also, the IEEE 118-bus system has 186 branches, ten ZIBs, and seven RBs in total. Furthermore, flow measurements are also shown.

Table 1. Statistics for test networks.

Test networks	# Lines	# ZIBs	ZIBs Position	# RBs	RBs Position	Flow Measurements
14- bus	20	1	7	1	8	1-2, 2-3, 6-11, 7-8, 10-11
57- bus	80	15	4, 7, 11, 21, 22, 24, 26, 34, 36, 37, 39, 40, 45, 46, 48	–	–	1-2, 1-15, 1-16, 1-17, 3-15, 4-5, 4-6, 4-18, 7-29, 8-9, 9-10, 10-12,10-51, 11-41, 11-43, 12-13, 14-46, 19-20, 20-21, 22-38, 23-24, 24-25, 24-26, 27-26, 28-27, 29-52, 30-31, 32-34, 34-35, 36-35, 38-37, 38-44, 38-48, 40-36, 41-42, 42-56, 47-46, 49-38, 51-50, 53-54
118- bus	186	10	5, 9, 30, 37, 38, 63, 64, 68, 71, 81	7	10, 73, 87, 111, 112, 116, 117	1-3, 5-6, 11-13, 16-17, 20-21, 22-23, 23-25, 27-28, 29-31, 34-43, 35-36, 41-42, 44-45, 46-48, 50-57, 51-52, 53-54, 56-58, 60-62, 65-66, 66-67, 68-81, 71-73, 75-118, 76-77, 77-82, 78-89, 86-87, 90-91, 95-96, 100-101, 114-115

Table 2 ssummarizes the OPPP solution for all test systems operating in their usual mode of operation. Here, a total of 4 PMUs, 17 PMUs and 32 PMUs were necessary to fully monitor the 14-bus, 57-bus, and 118-bus test systems, respectively. From Table 2, it is clear that the number of PMUs required to achieve full observability increases in direct proportion to the size of the power network. Furthermore, according to the existing study, the number of PMUs is in the region of 20 percent to 30 percent of the total number of buses that must be examined on a regular basis, which is a substantial reduction. In addition, as the network dimension increases, so does the computing time.

Table 2. OPPP solutions in the absence of ZIBs.

Test networks	# PMU	Positions of PMUs
14-bus	4	2, 6, 7, 9
57-bus	17	1, 4, 9, 20, 24, 27, 29, 30, 32, 36, 38, 41, 44, 46, 51, 54, 57
118-bus	32	3, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 40, 45, 49, 52, 56, 62, 64, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114

PMUs must be placed at optimal positions in order to achieve the absolute minimum number of PMUs necessary when ZIBs are present, as shown in Table 3. It should be noted that when the ZIB is taken into account in the simulation, the number of PMUs required for all buses reduces significantly. For example, in a 14-bus system, the number of PMUs necessary to achieve complete observability of the network under conventional operating conditions is 4, however with ZIB, only three PMUs are required.

Table 3. OPPP solution in the presence of ZIBs.

Test networks	# PMU	Position of PMUs
14-bus	3	2, 6, 9
57-bus	11	1, 6, 13, 19, 25, 29, 32, 38, 51, 54, 56
118-bus	28	3, 8, 11, 12, 17, 21, 27, 31, 32, 34, 37, 40, 45, 49, 52, 56, 62, 72, 75, 77, 80, 85, 86, 90, 94, 102, 105, 110

Table 2 shows the optimal amount of PMUs to use, as well as the optimal locations. However, in Table 4, the location of PMUs is optimized for the purpose of increasing the number of observable buses, that is, buses that can be measured by PMUs. Buses 2, 6, 7, 9, and 14 are the consequent PMU positions for 14-bus system. All PMU positions in the 14-bus system are the same as they are during normal operation. Whereas, bus 14 is the location of the additional PMU. Despite the fact that it appears that the optimization is done locally, that is not the case. It just so occurs to be at the most strategic positions. Similarly, for other test networks, the results are portrayed in Table 4 and Table 5 with absence and presence of ZIBs.

Table 4. OPPP solutions for maximum observability in the absence of ZIBs.

Test networks	# PMU	Position of PMUs
14-bus	5	2, 6, 7, 9, 14
57-bus	18	1, 6, 9, 15, 19, 22, 24, 28, 31, 32, 35, 38, 41, 47, 50, 53, 56, 57
118-bus	33	3, 5, 10, 11, 12, 17, 21, 25, 28, 34, 37, 40, 45, 49, 53, 56, 62, 63, 68, 70, 71, 77, 80, 85, 86, 90, 92, 96, 100, 105, 110, 114, 118

Table 5. OPPP solutions for maximum observability in the presence ZIBs.

Test networks	# PMU	Position of PMUs
14-bus	4	2, 4, 6, 9
57-bus	13	1, 3, 9, 13, 20, 27, 29, 31, 33, 36, 50, 53, 56
118-bus	29	3, 9, 12, 15, 19, 22, 27, 31, 32, 34, 42, 45, 49, 53, 56, 62, 65, 70, 75, 79, 85, 86, 90, 92, 96, 100, 105, 110, 118

The criticality of each PMU is assumed in the optimization problem investigated in this study. It will not be known whether or not the PMU measurement will be dropped until after the event has taken place. As a result, it is presumed that each PMU that has been installed has an equal chance of being unavailable. The comprehensive observability assessment of the 14-bus network is depicted in Table 6. The unobservable buses (while taking into account the malfunction of each of the single allocated PMU) are provided in this table, which has a total of 5 PMUs. Due to the obviousness of the suggested technique, a minimal number of buses are not observable, as was intended. It is also possible to see the redundancy of individual buses for the whole network. As can be noted from Table 6, even if the PMU on bus 14 is no longer operational, the network seems to be capable of achieving complete observability.

The observability of the 57-bus and 118-bus networks is illustrated in Tables 7 and 8, respectively. The number of buses seen by more than one PMU is clearly mentioned, despite the fact that no information is supplied concerning the redundancy of each bus. It can be observed from both tables that the influence of PMU failure on the observability is higher for bigger networks than for smaller networks. A few of the PMUs are extremely critical; consequently, the failure of those specific PMUs may lead to an increase in the number of unobservable buses. The PMU-equipped bus 17 was the most critical bus for the entire 118-bus

Table 6. Results of the observability test for the IEEE 14-bus system.

Failure of PMU-equipped bus	**Measurement of the redundancy for each bus**	**Buses**														**Buses that are not observable**	**Rank-wise vulnerability of PMU-equipped bus**
		1	2	3	4	5	6	7	8	9	10	11	12	13	14		
2	R_m	0	0	0	2	1	1	2	1	3	1	1	1	2	2	1, 2, 3	1
6	R_m	1	1	1	3	1	0	2	1	3	1	0	0	1	2	6, 11, 12	2
7	R_m	1	1	1	2	2	1	1	0	2	1	1	1	2	2	8	3
9	R_m	1	1	1	2	2	1	1	1	2	0	1	1	2	1	10	4
14	R_m	1	1	1	3	2	1	2	1	2	1	1	1	1	1	–	5

network. Whenever the PMU located at bus 17 fails to function, virtually 6 buses become unobservable simultaneously. Moreover, when a single PMU is inaccessible, the programmed technique detects the accessible PMUs so that the largest number of observable buses is attained.

Table 7. Results of the observability test for the IEEE 57-bus system.

Failure of PMU-equipped bus	**Buses that are not observable**	**Buses observed by multiple PMUs**	**Rank-wise Vulnerability of PMU-equipped bus**
1	2, 16, 17	15	3
6	4, 5, 6, 7	16	2
9	9, 10, 12, 55	14	1
15	3, 14, 45	14	4
19	18, 19, 20	17	5
22	21	14	13
24	24, 25, 26	16	6
28	27, 28, 29	17	7
31	30	15	14
32	33	14	15
35	35, 36	16	9
38	37, 44	13	10
41	43	14	16
47	46, 47	16	11
50	50, 51	16	12
53	52, 53, 54	17	8
56	40	14	17
57	39	16	18

Table 8. Results of the observability test for the IEEE 118-bus system.

Failure of PMU equipped bus	Buses that are not observable	Buses observed by multiple PMUs	Rank-wise vulnerability of PMU equipped bus
3	1	40	24
5	6, 8	39	15
10	9, 10	40	16
11	14	38	25
12	2, 7, 14, 117	39	2
17	15, 17, 18, 30, 31, 113	39	1
21	20, 21, 22	40	7
25	23, 25, 26	39	8
28	28, 29	39	17
34	19, 36, 43	39	9
37	33, 35, 38	36	10
40	41	37	26
45	44, 46	38	18
49	47, 48, 50, 51	36	3
53	52, 53	40	19
56	55, 56, 57, 58	39	4
62	60, 61, 62, 67	39	5
63	63, 64	39	20
68	65, 68, 116	39	11
70	24, 74	38	21
71	72, 73	38	22
77	78	37	27
80	79	34	28
85	83, 84, 88	38	12
86	87	38	29
90	90	39	30
92	93, 102	37	23
96	95	37	31
100	101	34	32
105	105, 107, 108	38	13
110	109, 110, 111, 112	40	6
114	32, 114, 115	40	14
118	118	39	33

Table 9 shows the comparative analysis for PMU placement in absence and presence of conventional measurements for maximum observability, and Table 10 provides the details of PMU placement considering the failure of PMU equipped bus.

It has been determined that the optimal number of PMUs is required for the full observability of the test system under typical operating circumstances. Depending on it, a new PMU can be installed into the network. The overall number of PMUs is optimized on a global scale and put at specified places, as shown in Tables 2-5 and Table 9. The number of PMUs must be more than or equivalent to conventional measures for accurate measurement. Since standard measurements are employed, the number of PMUs required is reduced by a significant amount compared to the previous estimates. In Tables 4-5 and Table 9, the number

Table 9. Comparison of the installation of PMU in presence and absence of conventional measurements with the goal of maximizing observability.

Test networks	**Installation of PMU**			
	Normal operation		*Considering Maximum observability*	
	Absence of conventional measurements	**Presence of conventional measurements**	**Absence of conventional measurements**	**Presence of conventional measurements**
14-bus	4	3	5	4
57-bus	17	11	18	13
118-bus	32	28	33	29

of PMUs and their positions can be compared, and it can be concluded that the highest number of buses are located in the most suitable location.

Table 10. OPPP solutions considering rank-wise failure of PMU-equipped bus for IEEE 14, IEEE 57 and IEEE 118 bus systems.

Test Systems	**Failure of PMU equipped bus**	**OPPP solutions**
14-bus	2	1, 4, 6, 7, 9
	6	1, 4, 6, 7, 9
	7	1, 4, 6, 7, 9
	9	1, 4, 6, 7, 9
	14	1, 4, 6, 7, 9
57-bus	1	1, 4, 6, 13, 20, 22, 25, 27, 29, 32, 36, 39, 41, 44, 47, 51, 54
	6	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	9	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	15	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	19	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	22	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	24	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	28	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	31	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	32	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	35	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	38	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	41	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	47	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	50	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	53	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	56	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
	57	1, 4, 8, 13, 20, 23, 27, 30, 32, 36, 39, 41, 44, 47, 51, 52, 55
118-bus	3	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114

5	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
10	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
11	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
12	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
17	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
21	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
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49	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
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56	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114

62	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
63	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
68	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
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71	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
77	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
80	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
85	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
86	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
90	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
92	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
96	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
100	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
105	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
110	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114

114	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114
118	1, 5, 9, 12, 15, 17, 21, 25, 28, 34, 37, 41, 45, 49, 53, 56, 62, 63, 68, 70, 71, 76, 79, 85, 86, 89, 92, 96, 100, 105, 110, 114

No limitation was established throughout the optimization process to prevent the PMU from being put at zero injection buses. Because of this, the suggested optimized approach allows PMU to be implemented even on zero-injection buses. Because each PMU is treated as crucial in this study, the optimization problem under consideration is a criticality problem. It will not be realized which PMU measurement will be malfunction until after the event has occurred, because the occurrence is unpredictable. As a result, it is presumed that each PMU that has been installed has an equal chance of being unavailable. The results shown in Tables 4 and 5 do not guarantee that the system will be fully observable in its entirety.

5. Conclusion

This article investigated the PMU placement strategy for situations in which there are only a finite number of accessible PMUs. The goal was to increase the system's observability to the maximum level. It was considered that there was a proportionally equivalent risk of failure or unavailability for each and every PMU that was put into operation. As a consequence, the position of the derived PMU assures that even if one of the PMUs fails, the other PMUs deployed will still be able to monitor the maximum possible number of buses. It was suggested in this work that the list of requirements should be expanded to include an extra maximal observability criterion. In addition, the proposed approach is essential for figuring out which PMU, out of all of the PMUs, is the most significant. The use of binary integer linear programming was necessary in order to arrive at a deterministic solution, and the optimization process was carried out on a global scale. This method may be employed to explore the observability of a system in an effective manner when PMUs are deployed in a sub-transmission network in a manner that is completely at random.

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