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Article

The solution of the economic dispatch problem via an efficient Teaching-Learning-Based Optimization method

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Abstract: This paper is concerned with the economic generation dispatch problem. It is a well-known fact that practical aspects of power plant equipment, as well as the objectives to be met, may result in a nonconvex, nondifferentiable model that poses difficulties to conventional mathematical programming methods. This paper proposes the use of metaheuristic Teaching-Learning-Based Optimization to overcome such difficulties. This metaheuristic is well known for requiring a few parameters and, most importantly, it does not require the tuning of problem-dependent parameters. The algorithm proposed in this work is parameter-free; that is, the few parameters required by the Teaching-Learning-Based Optimization method are set automatically based on the power system's data. In addition, the handling of constraints, such as generators' prohibited zones and the generator-load-loss power balance, is performed in a very efficient way. Simulation results are shown for power systems containing 3 to 40 generation units, and the results provided by the proposed method are shown and discussed based on comparisons with other metaheuristics and a mathematical programming technique.

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1. Introduction

The economic dispatch (ED) problem can be concisely defined as the setting of output powers from generating units to meet the demand at the minimum cost [1, 2]. The ED problem can be regarded as a subproblem of the unit commitment problem [2]. The latter involves deciding which generation units must be turned on to meet a certain demand. Hence, the ED problem involves determining the optimal operating points of the committed generation units. The fuel rates are important factors that allow the generation agents to achieve their highest possible efficiency levels [3]. The minimization of fuel costs eventually results in lower energy costs for the consumers.

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The basic formulation of the economic generation dispatch problem has been long proposed in the literature [4]. The ED constitutes a particular case of a broad set of optimal power flow (OPF) problems [5]. The OPF formulation consists in general of a non-linear, constrained, optimization problem as follows,

$$\begin{aligned} \min f(x,u) \\ \text{subject to}\, g(x,u) = 0, \\ h(x,y) \leq 0, \end{aligned} \tag{1}$$

where $f(x,u)$ is the objective function, x is the array of dependent or state variables, and u is the array of independent or control variables. $g(x,u)$ and $h(x,u)$ are respectively the set of equality and inequality constraints. Examples of control variables u are real power at generation buses, voltage magnitude at PV buses, transformer taps, and shunt (capacitive and inductive) elements. The state variable vector x may contain voltage magnitudes at load buses, reactive powers at generation buses, and transmission line power loadings, among others.

Several methods have been reported to solve the economic dispatch problem. In general, they can be broadly divided into two classes, namely (a) the mathematical programming methods and (b) the heuristics and metaheuristics. The mathematical programming methods assume that functions are differentiable or at least piecewise differentiable [2]. However, practical ED problems may require the inclusion of several physical characteristics of the generation units that result in non-differentiable functions such as the existence of prohibited generation zones. As a result, the ED model may be nonlinear and nonconvex, and thus, conventional programming methods may exhibit convergence difficulties and computational inefficiency. To overcome such problems, heuristics and metaheuristics appear to be promising alternatives because they can handle nonconvex problems in an easier and seamless manner.

Several metaheuristics have been proposed for solving the economic dispatch problem as the Genetic Algorithm (GA) [6, 7], Particle swarm optimization (PSO) [2], Teaching-Learning-Based optimization [8], Artificial immune system [9], Firefly algorithm [10], among others.

Also, optimization methods can be either deterministic or stochastic [11]. Deterministic algorithms always result in the same solution if the iterative process starts with the same input. On the other hand, stochastic algorithms provide different solutions even if the same input is provided. It is important to mention that those different solutions may be considered practically the same, given an accuracy threshold. It is not possible to guarantee that using those methods lead to the optimal solution, however, experience shows that they provide very good-quality solutions.

One of the main difficulties associated with metaheuristics is that several of them have parameters whose settings are problem dependent. For instance, PSO [2] requires the inertia weight factors and acceleration constants. The Firefly Algorithm (FA) [11] requires the light absorption coefficient and attractiveness. Finally, the mutation rate, crossover probability, and the selection method are required in the GA [12].

The metaheuristic Teaching-Learning-Based Optimization (TLBO) was first proposed by Rao, Savsani, and Vakharia [12]. TLBO is a nature-based algorithm, particularly, a population-based algorithm. TLBO does not require any problem-dependent parameter to be tuned [13] other than the population size and number of iterations [14]. This very interesting feature makes its implementation much simpler. According to [14], TLBO is considered an algorithm-specific, parameter-less algorithm.

A review paper [13] provided a survey on the later developments in the TLBO algorithm and its applications, as well as a description of the application fields. A modified TLBO algorithm was proposed in [8]for solving the ED. Its main characteristics were the inclusion of a linearly variable teacher factor and a tournament-based procedure for picking individuals. The basic TLBO algorithm in [14] included an elitism procedure, thus preserving the best individuals along the iteration process. An improved TLBO algorithm was proposed in [15], by considering that students also learn during tutorial hours by discussing with their classmates or even by a discussion with the teacher. In addition, students were sometimes self-motivated and tried to learn by themselves. Reference [16] shows a comparison of metaheuristics GA, PSO, Simulated Annealing (SA), Exchange Market Algorithm (EMA), and TLBO for solving the ED problem, and TLBO showed superior performance. An extensive comparison of metaheuristics for solving the optimal power flow problem was presented in [5]. Once again, TLBO showed to be very efficient.

TLBO presents many advantages, such as (a) fewer parameters, (b) simple algorithm, (c) easy to understand, (d) fast solution speed, (e) high accuracy, and (f) good convergence ability [13]. Being very flexible, TLBO allows different variations and improvements, which is very interesting and even desired in the optimization field. Because of its interesting features, TLBO has also been used for solving several power-system-related problems besides the ED. Some examples are the optimal capacitor placement in distribution systems [17], and distribution systems reconfiguration [18], among others.

The main goal of this paper is to apply an efficient version of the Teaching-Learning-Based Optimization (TLBO) metaheuristic for solving the ED problem. The main contributions of this paper are twofold:

- TLBO was implemented such that no system parameters must be specifically defined. They are automatically set according to the system's characteristics.
- The inequality constraints are dealt with such as to avoid the use of penalty factors and therefore the distortion of the objective function, making the convergence smoother.

The simulation results show that the proposed method leads to excellent results compared with those in the literature. In addition, TLBO is conceptually simple and computationally efficient. This paper constitutes a step forward in the development of optimization methods applied to power systems. The ability to solve real-life problems by overcoming problems resulting from non-convexities and non-differentiabilities is crucial.

2. Economic dispatch model

One of the requirements for an efficient power system operation is to make sure that the power is generated at the minimum cost. In practice, the overall minimum cost cannot be obtained due to operational constraints, which must be considered. In this case, the idea is to generate power at the least cost possible. This process is known as the economic dispatch problem, and its main features will be described in this section.

2.1. Objective function

As mentioned earlier, the ED problem consists of specifying the output powers from generation units to meet the load at the minimum cost. Each thermal generation unit i is usually associated with a quadratic cost function C_i which depends on the output power P_i [1]. For a power system with N generation units, the total generation cost function C_t is given by,

$$C_t = \sum_{i=1}^{N} C_i(P_i) = \sum_{i=1}^{N} (a_i + b_i P_i + c_i P_i^2), \tag{2}$$

where coefficients a_i, b_i, and c_i are previously known.

2.2. Equality constraint

The power generated by the several generation units must supply the load and the transmission power losses. This requirement is known as the power balance constraint, and it is given by,

$$\sum_{i=1}^{N}(P_i) - P_D - P_L = 0, \tag{3}$$

where P_D is the total power demand and P_L corresponds to the total transmission power losses.

A simple, widespread way of representing the transmission power losses is by considering them a quadratic function of the generation output powers [1, 2, 16], as,

$$P_L = B_{00} + \sum_{i=1}^{N}(B_{0j}P_i) = \sum_{i=1}^{N}\sum_{j=1}^{N}(P_i B_{ij} P_j), \tag{4}$$

where B_{00}, B_{0j}, and B_{ij} are known as the B coefficients.

2.3. Inequality constraint – generation limits

The output power of generation units must lie within the range,

$$P_i^l \leq P_i \leq P_i^u, i = 1, ..., N, \tag{5}$$

where P_i^l and P_i^u are respectively the lower and upper bounds associated to generation unit i.

2.4. Inequality constraint – prohibited operating zones

Prohibited zones are due to the steam valve operating or vibration in a shaft bearing [6]. The prohibited zones associated with generation unit i are modelled as,

$$P_{i,j}^l \leq P_i \leq P_{i,j}^u, j = 1, ..., NP_i, \tag{6}$$

where NP_i is the number of prohibited zones of generation unit i. $P_{i,j}^l$ and $P_{i,j}^u$ are the lower and upper limits of prohibited zone j, respectively. Figure 1 shows an example cost curve for generation unit i as a function of its generated power. Note the presence of two prohibited zones within the allowed power range as described in Sec. 2.3.

2.5. Proposed ED model

The ED model proposed this paper is,

$$\begin{gathered} \min (2) \\ \text{subject to} (3), (5), (6), \end{gathered} \tag{7}$$

the objective Eq. (2) and the power balance constraint Eq. (3) are quadratic. It is worth pointing out once more that the presence of prohibited zones implies in difficulties to conventional mathematical programming methods.

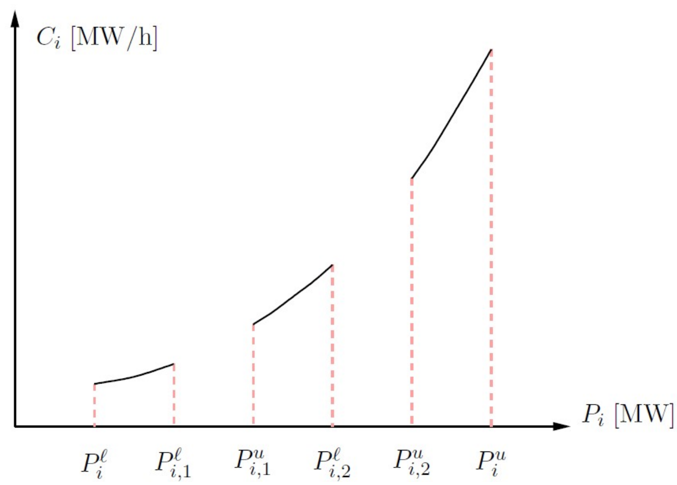

Figure 1. Example cost curve for generation unit i. Adapted from [6].

2.6. Other aspects of ED models

The basic model of the economic dispatch is theoretically simple; however, it may become increasingly complex due to the size of the problem, the coordination the different characteristics and operating costs due to different generation technologies and sources, the variations in load over daily and seasonal cycles, and the need to operate the system reliably, abiding by transmission line operating limits [19]. In addition, the security-constrained economic dispatch adds more difficulties to the problem since it must consider the possibility outages (contingencies), either in the generation or the transmission systems.

Still, according to [19], the economic dispatch has gotten more complex because of the incorporation of public policy changes, technological innovation, and the ever-increasing penetration of stochastic, intermittent generation, such as wind and solar generation, as well as energy storage.

Several other practical aspects can be included in the ED model. It is worth noting that the inclusion of such aspects does not affect the performance of the metaheuristic algorithm proposed in this paper. Some of those aspects are included here for the sake of example, without the intention of being comprehensive.

It is possible to include the valve-point effects, which appears in the objective function, by including an additional term to (OC) [7, 16, 20], resulting in,

$$C_t' = \sum_{i=1}^{N}(a_i + b_iP_i + c_iP_i^2) + |e_i \cdot sin\,[f_i \cdot (P_{i,min} - P_i)]\,| \tag{8}$$

where e_i and f_i are coefficients related to the valve loading of generation i.

Also, [20] modelled the fuel cost as a cubic function and discussed the inclusion of other aspects to the objective function, such as the cost of emission of pollutants and fuel limitation.

A constraint related to imposing limits to transmission line power flows are represented as,

$$|P_k| \leq P_k^{lim} \qquad k = 1, ..., L, \tag{9}$$

where L is the number of transmission lines.

The ramp rate limit constraint is related to maximum generation changes between time periods, being given by,

$$-Dr_i \leq P_{i,t} - P_{i,(t-1)} \leq UR_i, \qquad i = 1, ..., N, \tag{10}$$

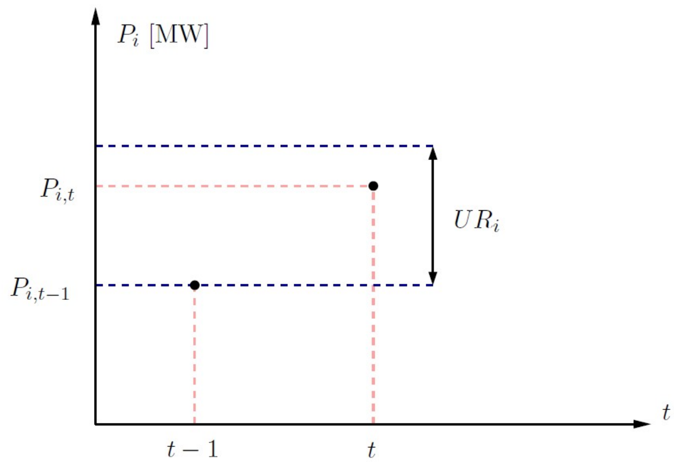

Figure 2. Illustration of ramp limits for generation unit i. Adapted from [6].

where DR_i and UR_i are respectively the down-ramp and up-ramp limits from generation unit i. $P_{i,t}$ and $P_{i,(t-1)}$ are the generation output powers of generation unit i at two consecutive time instants. Figure 2 illustrates the case where $P_{i,t} > P_{i,(t-1)}$. Note that $P_{i,t}$ must lie within the range delimited by UR_i.

It is worth noting that the aspects mentioned in this section can be included in the model and solved by metaheuristics despite their characteristics as far as linearity and convexity are concerned.

3. Teaching-Learning-Based Optimization

The algorithm of the TLBO implemented in this paper was based on [12] and [21]. Both the teacher and student phases were applied to all individuals. Also, the removal of duplicate individuals was not included. As a result, the number of evaluations of the objective function is deterministic, given by,

$$\text{Eval} = (2 \cdot IT + 1) \cdot N_p, \tag{11}$$

where IT is the number of iterations and N_p is the population size.

3.1. Basic algorithm

The following pseudo-code (Algorithm 1) shows the basic steps of the algorithm proposed and implemented in this paper.

3.2. Implementation details and comments

The following comments regarding the algorithm are important for its implementation.

- Both teacher and student phases consist of greedy selection processes, where a new individual (solution candidate) solution is accepted whenever it is better than the current one.
- The population size is defined automatically in terms of the number of generation units N, as,

$$N_p = 10 \cdot N, \tag{12}$$

  therefore, the proposed algorithm is parameter-free.
- The teaching factor T_F can be either set as a constant [21] or as a random value [12]. In this paper, T_F was set according to [12].

Algorithm 1: Teaching-Learning-Based Optimization Algorithm.

1. Generate the initial Population $X_i, i = 1, \ldots, N_p$

Teacher phase:;
2. Compute the mean individual $M = \text{mean}(X)$;
3. Choose the Teacher T;
4. Compute direction $\Delta = r \cdot (T - T_F \cdot M)$, where r is a random number in the range $[0, 1]$ and T_F is the teaching factor randomly chosen as either 1 or 2;
5. **for** *each individual* $i, i = 1, \ldots, N_p$ **do**
    a. Obtain new individual $X_i' = X_i + \Delta$;
    b. **if** X_i *is better than* X_i' **then**
        Maintain X_i in the population;
    else
        $X_i \leftarrow X_i'$;
    end
  end
end

Student phase:;
6. **for** *each individual* $X_i, i = 1, \ldots, N_p$ **do**
    a. Choose an individual X_j randomly;
    b. **if** X_i *is better than* X_j **then**
        $Better = X_i$ and $Worse = X_j$;
    else
        $Better = X_j$ and $Worse = X_i$;
    end
  end
    c. Compute $Diff = Better - Worse$;
    d. Obtain a new individual $X_i' = X_i + r \cdot Diff$;
    e. **if** X_i *is better than* X_i' **then**
        Maintain X_i in the population;
    else
        $X_i \leftarrow X_i'$;
    end
  end
end
7. Stopping criterion was met? If so, stop. Else, go back to step 2;

- The stopping criterion adopted in this paper is based on the evolution of the objective function. If the objective function remains the same for $(10 \cdot N)$ consecutive iterations, it is assumed that its optimal value has been reached and the process is interrupted.
- Whenever new individuals are generated, such as in steps 1, 5(a), and 6(d), it is necessary to verify whether they meet the equality and inequality constraints. In other words:
  - The power balance constraint Eq. (3) must be met.
  - The generated powers must be within the lower-upper range Eq. (5), and outside the prohibited zone regions Eq. (6).

  The way to deal with these inequalities is described in detail in the next section.

4. Dealing with constraints

Consider the following mathematical programming problem,

$$\begin{array}{rlr} \min & f(u) & \\ \text{subject to} & g_i(u) \geq 0 & i = 1, \dots, NI \\ & h_j(u) = 0 & j = 1, \dots, NE \\ & u_k^l \leq u_k \leq u_k^u & k = 1, \dots, N, \end{array} \tag{13}$$

where f is the objective function, and g and h are respectively the sets of inequality and equality constraints. u is the array of decision variables, bound by lower and upper limits u^l and u^u. N, NE, and NI are respectively the numbers of decision variables, equality constraints, and inequality constraints.

According to [22], the conventional strategies of handling constraints in an optimization problem can be broadly classified as Eq. (1) inclusion penalty functions, Eq. (2) decoders, Eq. (3) special operators, and Eq. (4) separation of objective function and constraints. The latter strategy is used in this paper. As mentioned earlier, whenever new individuals are generated, it is necessary to verify whether they meet the equality and inequality constraints. This verification is described in detail ahead.

4.1. Equality constraints

A conventional way of dealing with equality constraints is by squaring and adding them to the objective function using a penalty factor, resulting in,

$$F(u) = f(u) + \mu \cdot h_j^2(u), \tag{14}$$

where the penalty factor μ is usually a large number. According to [23], the penalty factor has several drawbacks, such as (a) it is system-dependent, therefore, the user must search for the best factor through a trial-and-error procedure whenever any parameter undergoes any change; (b) its value influences significantly the solution of the problem; and (c) the penalty factor μ causes a distortion of F and, depending on its value, this distortion may lead to artificial, local optimal solutions.

To cope with this problem, in this paper the equality constraints in Eq. (13) are replaced by,

$$|h_j(u)| \leq \varepsilon, \tag{15}$$

where ε is a small, positive threshold value.

It is a common practice to add the constraints to the objective function with the inclusion of penalty factors. In this paper, the constraints are treated separately, following the principles presented in [23]. Whenever a candidate solution is evaluated, the following rules apply.

- Any feasible solution is preferred to any infeasible solution.
- Among two feasible solutions, the one having a better objective function value is preferred.
- Among two infeasible solutions, the one having smaller constraint violation is preferred.

It will be shown that by handling the constraints described above, the drawbacks associated with the use of penalty factors mentioned earlier can be overcome. Setting ε in Eq. (15) is significantly easier than that of μ in Eq. (14), and its value does not significantly affect the results. This constraint-handling technique has been used in several studies [14]. Considering that the model in Eq. (7) adopted in this study, there is only one equality constraint represented by Eq. (3). Parameter ε was set to 0.05 MW, which is the same for any power system.

4.2. Inequality constraints

In this paper, the inequality constraints included in Eq. (7) are:

- The lower and upper power bounds of the generation units, and
- The prohibited power generation regions.

In this paper the following, simple rule is used to consider inequality constraints: "if a generation unit violates a certain limit, its output value is set to the closest violated limit." As an example, assume a generation unit with lower and upper limits corresponding to 100 MW and 300 MW, respectively. In addition, we consider a prohibited zone $[140 - 170]$ MW. If, in a certain iteration, the generation is set to 320 MW (violation of the upper limit), it is reset to 300 MW. If the generation is set to 160 MW (within the prohibited zone), it is reset to 170 MW.

5. Simulation results

This section presents the simulation results for power systems with 3, 6, 15, and 40 generation units. These systems are well known in the literature and their respective data are widely available. Therefore, the results of the proposed method can be compared with those available in the literature. The results provided by the proposed method were compared with those of other metaheuristics and mathematical programming methods. The latter is successive quadratic programming, implemented using the SQP function from Octave [24]. SQP is assumed to provide the exact solution, and it is taken as a reference for comparison with other methods. Each simulation was run 100 times, and the averages and standard deviations are shown.

5.1. 3-unit system

The data from the 3-unit system was taken from [1]. The load for this system was set to 850 MW. In this case the generators do not present prohibited zones, therefore, model Eq. (7) does not include Eq. (6). Table 1 presents a comparison of the results obtained with TLBO with those provided by running function SQP from Matlab. Function SQP (Sequential quadratic programming) is an iterative method for constrained nonlinear optimization problems.

The results provided by TLBO are very close to those obtained by SQP, since the global optimum operating point provided by SQP lies within the range of values provided by TLBO (average $\pm$ std deviation). Note also that the 3-unit system is considered small, since it has only three decision variables. Also, since this system does not present any prohibited zones, there are not non-convexities in the model.

	Iterations	P_1 [MW]	P_2 [MW]	P_3 [MW]	$P_{\mathbf{loss}}$ [MW]	C_t [$]
Average	52.4	438.5	292.7	132.7	13.9	8,335.50
Std Deviation	19.8	36.4	34.7	23.1	2.24	18.7
SQP	-	435.2	300.0	130.7	15.8	8,344.60

Table 1. 3-unit system – simulation results provided by TLBO and [1].

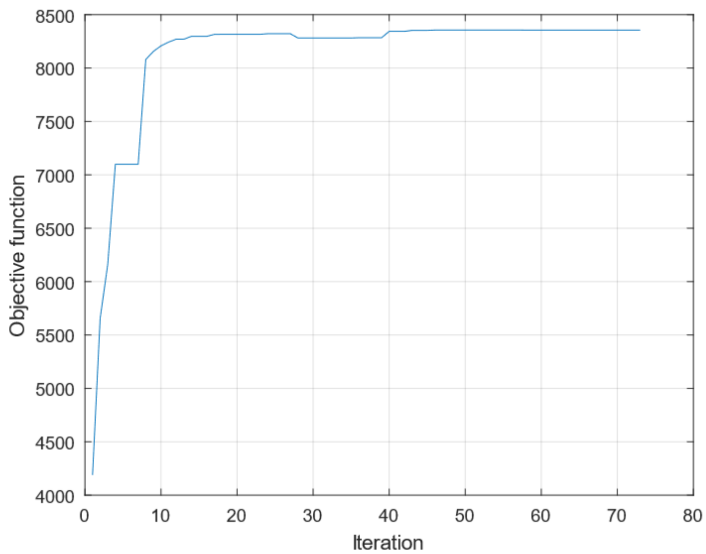

Figure 3. 3-unit system – evolution of the objective function.

Figure 3 shows the evolution of the value of the objective function along the iterative process. The presence of the equality constraint Eq. (3) leads the objective function to increase as the iterative process progresses. In the first iterations, the individuals may have lower generation costs, however, the constraints are violated. Later on, the situation is reversed, and TLBO moves toward the optimal solution through feasible individuals.

5.2. 6-unit system

The complete data of the 6-unit system, including the cost coefficients, loss coefficients, generation limits, and prohibited zones was taken from [2]. The system's load was set to $1,263$ MW. Table 2 shows the results obtained by the proposed method.

A comparison of the best results obtained by the proposed TLBO, as well as by PSO and GA [2] are shown in Table 3. Column SQP is also included for comparison purposes.

Table 4 shows the average and best total generation costs provided by the proposed TLBO, as well as by PSO and GA [2], by modified algorithms of PSO [25, 26]. According to [2], PSO and GA were run 50 times. The proposed TLBO was run 100 times. Row SQP is also included for comparison purposes.

Table 2 to Table 4 clearly show that TLBO performed very well in comparison with the other metaheuristics, outperforming the other methods as far as the average and best generation costs, as compared with SQP.

Figure 4 shows the evolution of the values of the objective function for the 6-unit system. The general behavior of the objective function, in this case, is like the one from Figure 3.

	Average	**std desviation**
Iterations	68.9	20.1
P_1 [MW]	456.3	19.1
P_2 [MW]	173.3	17.4
P_3 [MW]	261.9	16.1
P_4 [MW]	131.1	14.1
P_5 [MW]	163.3	16.1
P_6 [MW]	89.2	15.7
Total [MW]	1,275.0	0.7
P_{loss} [MW]	12.0	0.7
C_t [$]	15,430.00	11.7

Table 2. 6-unit system – simulation results provided by TLBO.

	PSO [2]	**GA** [2]	**SQP**	**TLBO**
P_1 [MW]	447.5	474.8	450.2	457.0
P_2 [MW]	173.3	178.6	173.7	160.0
P_3 [MW]	263.5	262.2	258.4	269.4
P_4 [MW]	139.1	134.3	138.2	128.0
P_5 [MW]	165.5	151.9	163.6	163.1
P_6 [MW]	87.1	74.2	90.9	95.6
Total [MW]	1,276.0	1,276.0	1,275.1	1,273.0
P_{loss} [MW]	13.0	13.0	12.1	10.0
C_t [$]	15,450.00	15,459.00	15,419.00	15,393.00

Table 3. 6-unit system – comparison of best results.

	Average generation cost [$]	**Best generation cost** [$]
PSO [2]	15,454.00	15,450.00
GA [2]	15,469.00	15,459.00
MIPSO [25]	15,445.10	15,442.98
QPGPSO-W [26]	15,448.45	15,440.58
TLBO	15,430.00	15,393.00
SQP	15,419.00	

Table 4. 6-unit system – comparison of average and best results.

5.3. 15-unit system

The data from this system, including the cost coefficients, loss coefficients, generation limits, and prohibited zones was taken from [2]. The system's load is $2,630$ MW. Table 5 contains the simulation results obtained by TLBO.

A comparison of the best results provided by TLBO with those provided by PSO and GA [2] are shown in Table 6. Once more, column SQP was included for comparison purposes.

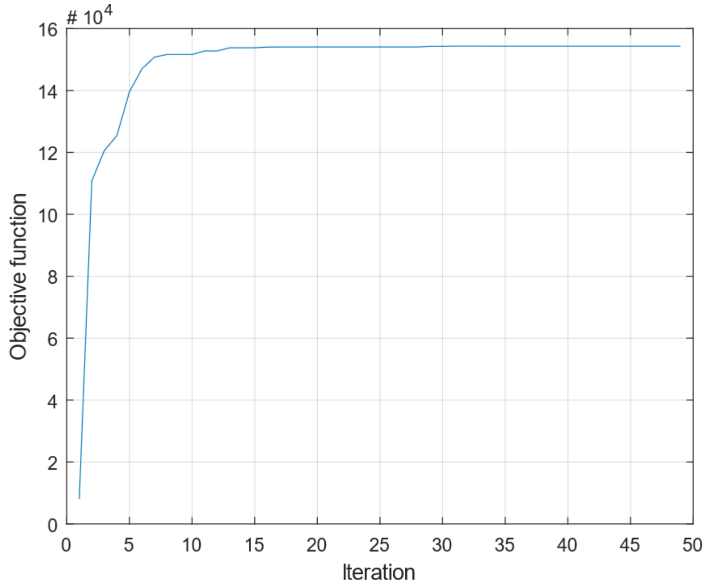

Figure 4. 6-unit system – evolution of the objective function.

	Average	**Std Deviation**
P_1 [MW]	401.4	71.5
P_2 [MW]	402.5	72.4
P_3 [MW]	119.0	63.2
P_4 [MW]	115.7	20.5
P_5 [MW]	322.2	27.0
P_6 [MW]	427.0	108.8
P_7 [MW]	437.8	53.6
P_8 [MW]	88.2	44.8
P_9 [MW]	60.5	43.2
P_{10} [MW]	81.6	41.9
P_{11} [MW]	50.1	51.2
P_{12} [MW]	60.1	23.0
P_{13} [MW]	36.7	22.1
P_{14} [MW]	28.9	15.5
P_{15} [MW]	30.7	14.3
iterations	161.1	71.5
Total [MW]	2,662.3	3.9
P_{loss} [MW]	32.3	3.9
C_t [$]	32,836.1	105.2

Table 5. 15-unit system – simulation results provided by TLBO.

Table 7 shows the average and best generation costs provided by TLBO, as well as by PSO and GA [2], by MIPSO [25], and by QPGPSO-W [26]. Again, TLBO was run 100 times, while PSO and GA were run 50 times [2]. Row SQP is also included for comparison purposes.

Table 5 to Table 7 show that TLBO performed very well as compared to the other methods and to SQP. The results provided by TLBO for this system stand among the best shown in Table 7.

	PSO	**GA**	**SQP**	**TLBO**
P_1 [MW]	439.1	415.3	455.0	425.7
P_2 [MW]	408.0	359.7	455.0	455.0
P_3 [MW]	119.6	104.4	130.0	130.0
P_4 [MW]	130.0	75.0	130.0	130.0
P_5 [MW]	151.1	380.3	235.3	233.6
P_6 [MW]	460.0	426.8	460.0	460.0
P_7 [MW]	425.6	341.3	465.0	465.0
P_8 [MW]	98.6	124.8	60.0	60.0
P_9 [MW]	113.5	133.1	25.0	25.9
P_{10} [MW]	101.1	89.3	30.0	101.4
P_{11} [MW]	33.91	60.06	75.0	38.41
P_{12} [MW]	79.96	50.00	80.0	50.50
P_{13} [MW]	25.00	38.77	25.0	27.02
P_{14} [MW]	41.41	41.94	15.0	18.78
P_{15} [MW]	35.61	22.64	15.0	36.51
Total [MW]	2,662.4	2,668.4	2,655.4	2,657.7
P_{loss} [MW]	32.43	38.28	25.4	27.69
C_t [$]	32,858.00	33,113.00	32,532.00	32,636.00

Table 6. 15-unit system – comparison of the best results.

	Average generation cost [$]	**Best generation cost** [$]
PSO [2]	33,039.00	32,858.00
GA [2]	33,228.00	33,113.00
MIPSO [27]	32,745.00	32,697.54
QPGPSO-W [25]	32,589.54	32,548.19
TLBO	32,836.08	32,636.00
SQP	32,532.00	

Table 7. 15-unit system – comparison of average and best results.

Figure 5 shows the evolution of the values of the objective function along the iterative process. Again, the behaviour of the objective function in this case is similar to the ones shown in Figure 3 and Figure 4.

5.4. 40-unit system

The complete data, including the cost coefficients and generation limits, can be found in [27]. The demand of the 40-unit system is $10,550$ MW. Table 8 shows the results obtained by TLBO after 100 runs.

	Generation cost [$]
Minimum	146,900.00
Average	151,786.20
Maximum	164,810.00
Std deviation	3,505.10

Table 8. Simulation results for the 40-unit system.

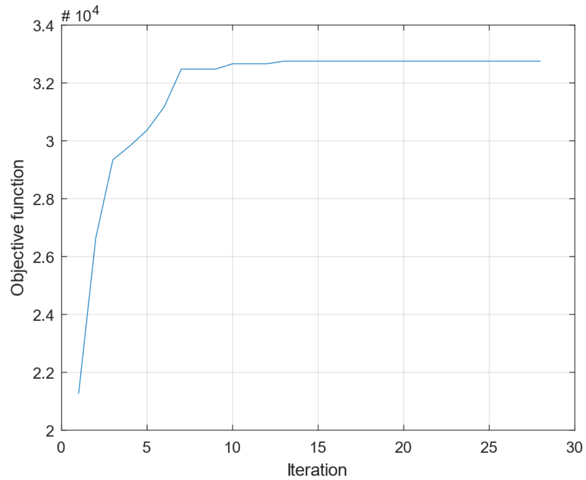

Figure 5. 15-unit system – evolution of the objective function.

Table 9 shows the average output powers for the generating units for a load of $10,550$ MW.

	Average [MW]	Std. deviation [MW]		Average [MW]	Std. deviation [MW]
P_1 [MW]	64.92	14.32	P_{21} [MW]	535.39	21.27
P_2 [MW]	101.95	18.95	P_{22} [MW]	537.43	24.14
P_3 [MW]	167.58	30.36	P_{23} [MW]	532.61	25.06
P_4 [MW]	33.29	6.73	P_{24} [MW]	531.32	28.03
P_5 [MW]	34.24	6.04	P_{25} [MW]	530.86	26.88
P_6 [MW]	119.32	25.25	P_{26} [MW]	529.55	32.3
P_7 [MW]	279.69	33.06	P_{27} [MW]	535.1	25.37
P_8 [MW]	272.33	41.59	P_{28} [MW]	47.27	33.62
P_9 [MW]	281.13	24.76	P_{29} [MW]	48.58	34.89
P_{10} [MW]	274.62	36.45	P_{30} [MW]	46.23	31.99
P_{11} [MW]	349.7	35.41	P_{31} [MW]	51.34	17.73
P_{12} [MW]	354.26	29.37	P_{32} [MW]	49.2	19.27
P_{13} [MW]	475.52	32.6	P_{33} [MW]	47.68	17.85
P_{14} [MW]	478.26	34.41	P_{34} [MW]	49.8	17.57
P_{15} [MW]	482.45	23.74	P_{35} [MW]	37.47	15.17
P_{16} [MW]	476.11	32.28	P_{36} [MW]	42.44	14.67
P_{17} [MW]	482.16	26.93	P_{37} [MW]	42.42	14.62
P_{18} [MW]	485.96	22.23	P_{38} [MW]	42.75	13.7
P_{19} [MW]	478.9	33.54	P_{39} [MW]	42.01	12.84
P_{20} [MW]	534.68	25.41	P_{40} [MW]	43.53	13.58

Table 9. Output powers for the 40-unit system.

Figure 6 shows a comparison of the results obtained by the proposed TLBO with those obtained by CSO, PSO, and GA [27]. The figure allows a visual comparison among the output generations provided by each method. By taking one generation unit at a time, it is possible to see that all four methods provide compatible, close results.

The average total generation and costs provided by CSO, PSO, and GA [27], as well as by the proposed TLBO are shown in Table 10. Note that the method adopted in this paper for handling the equality

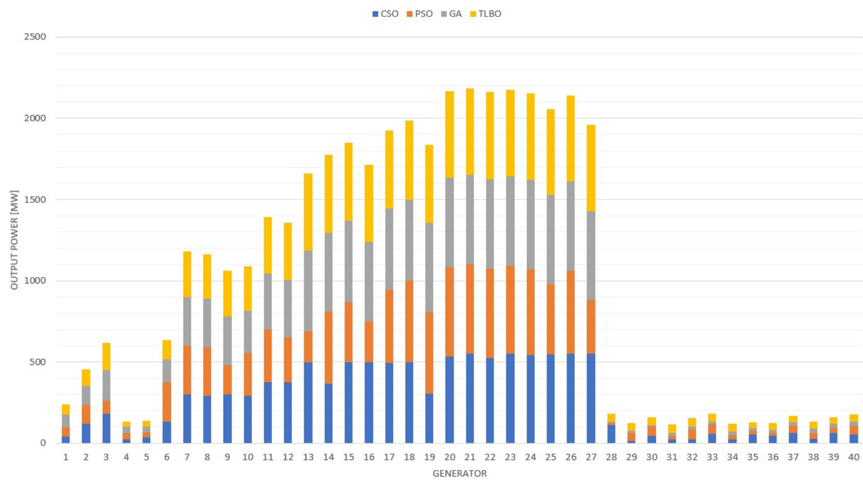

Figure 6. 40-unit system – Comparison of TLBO with CSO, PSO, and GA.

constraints allows a very good precision without creating numerical problems. Row SQP has been included for comparison purposes.

	Total cost [$/year]	Total generation [MW]
CSO [27]	133,438.27	10,549.91
PSO [27]	134,237.31	9,401.17
GA [27]	144,893.23	10,549.88
TLBO (average)	146,035.10	10,550.00
TLBO (best)	144,750.00	10,550.00
SQP	144,740.00	

Table 10. 40-unit system – comparison of TLBO, PSO, and GA.

It is not clear in [27] whether the results are the best ones, or average values, the reason why both the average and best results provided by TLBO were presented. Also, the total costs provided by TLBO are compatible with those provided by GA and SQP; however, they are larger than those provided by CSO and PSO. It seems odd that CSO and PSO show similar costs; however, the sum of all generations, as shown in [27], is quite different. In particular, the total generation reported for PSO is more than $1,000$ MW shorter than the specified value. The evolution of the objective function value is shown in Figure 7. Again, its general behavior is similar to that of previous ones.

6. Discussion

The simulation results presented in Section 5 clearly indicate the excellent characteristics of TLBO. In all simulations, TLBO provided the best or close-to-best results. All metaheuristics are based on a random initial population and random exploration and exploitation search. However, they may provide different results depending on the manner in which the searches are defined. In this work no changes in the TLBO algorithm were performed to adapt to the particular problem (ED). The idea of automatically defining the number of individuals in the population and handling constraints may be applied to any other optimization problem. Therefore, TLBO proved to have excellent potential for solving the economic dispatch problem and any other problem in the power system area.

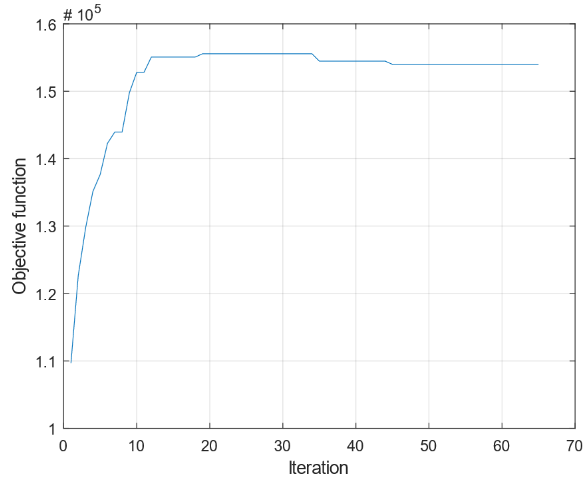

Figure 7. Evolution of the objective function for the 40-unit system.

7. Conclusions

This paper tackled the problem of economic generation dispatch. Even though this problem is well known, its formulation may result in a nonlinear, nonconvex model, which may pose numerical difficulties to mathematical programming methods. This work showed that metaheuristic TLBO is a simple and efficient method for solving ED. The automatic setting of its parameters, regardless of the system, is an important feature of the proposed method, and has been shown to be very effective. In addition, dealing with these constraints did not result in numerical problems, which are commonly found when penalty methods are used. In contrast, the iterative process was smooth, and high-quality results were obtained successfully. The simulation results obtained using the proposed method were compared with those reported in the literature. Also, simulation results from a mathematical programming method based on successive quadratic programming were also shown for comparison. The proposed method exhibited excellent performance. The use of TLBO to solve the ED problem has now expanded. The next steps consist of including actual aspects into the model, such as the effect of valve points and the minimization of the greenhouse effects, as described in Sec. 3.6. Also, the active power losses and power balance are more appropriately represented by the power flow equations. These aspects would result in a more realistic ED model and more precise representation of the electric system. The inclusion of nonlinear terms regarding the valve point effect would not significantly change the performance of TLBO because one of the strong characteristics of metaheuristics is their capability to handle nonlinearities and nonconvexities. The use of power flow equations does not at all affect the performance of TLBO, because its idea is to represent power losses in a more precise way and to automatically meet the generation-load balance (constraint).

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Appendix – Systems' data

3-unit generation system [1]

Gen	a	b	c	$P_{\mathbf{min}}$ [MW]	$P_{\mathbf{max}}$ [MW]
1	561	7.92	0.001562	150	600
2	310	7.85	0.00194	100	400
3	78	7.97	0.00482	50	200

Table 11. Data from the 3-unit generation system.

$$B_{00} = 0$$

$$B_{0j} = \begin{bmatrix} 0 & 0 & 0 \end{bmatrix}$$

$$B_{ij} = \begin{bmatrix} 0.00003 & 0 & 0 \\ 0 & 0.00009 & 0 \\ 0 & 0 & 0.00012 \end{bmatrix}$$

6-unit generation system [2]

Gen	a	b	c	$P_{\mathbf{min}}$ [MW]	$P_{\mathbf{max}}$ [MW]	P^l [MW]	P^u [MW]
1	240	7	0.007	100	500	210	240
2	200	10	0.0095	50	200	90	110
3	220	8.5	0.009	80	300	150	170
4	200	11	0.009	50	150	80	90
5	200	10.5	0.008	50	200	90	110
6	190	12	0.0075	50	120	75	85

Table 12. Data from the 6-unit generation system.

$$B_{00} = 0.056$$

$$B_{0j} = \begin{bmatrix} -0.3908 & -0.1297 & 0.7047 & 0.0591 & 0.2161 & -0.6635 \end{bmatrix} \times 10^{-3}$$

$$B_{ij} = \begin{bmatrix} 0.0017 & 0.0012 & 0.0007 & -0.0001 & -0.0005 & -0.0002 \\ 0.0012 & 0.0014 & 0.0009 & 0.0001 & -0.0006 & -0.0001 \\ 0.0007 & 0.0009 & 0.0031 & 0 & -0.001 & -0.0006 \\ -0.0001 & 0.0001 & 0 & 0.0024 & -0.0006 & -0.0008 \\ -0.0005 & -0.0006 & -0.001 & -0.0006 & 0.0129 & -0.0002 \\ -0.0002 & -0.0001 & -0.0006 & -0.0008 & -0.0002 & 0.015 \end{bmatrix}$$

Gen	a	b	c	P_{min} [MW]	P_{max} [MW]
1	671	10.1	0.000299	150	455
2	574	10.2	0.000183	150	455
3	374	8.8	0.001126	20	130
4	374	8.8	0.001126	20	130
5	461	10.4	0.000205	150	470
6	630	10.1	0.000301	135	460
7	548	9.8	0.000364	135	465
8	227	11.2	0.000338	60	300
9	173	11.2	0.000807	25	162
10	175	10.7	0.001203	25	160
11	186	10.2	0.003586	20	80
12	230	9.9	0.005513	20	80
13	225	13.1	0.000371	25	85
14	309	12.1	0.001929	15	55
15	323	12.4	0.004447	15	55

Table 13. Data from the 15-unit generation system.

Gen	Prohibited zones [MW]
2	[185 225] [305 335] [420 450]
5	[180 200] [305 335] [390 420]
6	[230 255] [365 395] [430 455]
12	[30 40] [55 65]

Table 14. Data from the 15-unit generation system (cont.).

15-unit generation system [2]

$$B_{00} = 0.0055$$

$$B_{0j} = [-0.0001 \quad -0.0002 \quad 0.0028 \quad -0.0001 \quad 0.0001 \quad -0.0003 \quad -0.0002 \quad -0.0002 \quad 0.0006 \quad 0.0039 \quad -0.0017 \quad 0 - 0.0032 \quad 0.0067 \quad -0.0064]$$

$$B_{ij} = \begin{bmatrix}
0.0014 & 0.0012 & 0.0007 & -0.0001 & -0.0003 & -0.0001 & -0.0001 & -0.0001 & -0.0003 & 0.0005 & -0.0003 & -0.0002 & 0.0004 & 0.0003 & -0.0001 \\
0.0012 & 0.0015 & 0.0013 & 0.0000 & -0.0005 & -0.0002 & 0.0000 & 0.0001 & -0.0002 & -0.0004 & -0.0004 & 0 & 0.0004 & 0.0010 & -0.0002 \\
0.0007 & 0.0013 & 0.0076 & -0.0001 & -0.0013 & -0.0009 & -0.0001 & 0.0000 & -0.0008 & -0.0012 & -0.0017 & 0 & -0.0026 & 0.0111 & -0.0028 \\
-0.0001 & 0.0000 & -0.0001 & 0.0034 & -0.0007 & -0.0004 & 0.0011 & 0.0050 & 0.0029 & 0.0032 & -0.0011 & 0 & 0.0001 & 0.0001 & -0.0026 \\
-0.0003 & -0.0005 & -0.0013 & -0.0007 & 0.0090 & 0.0014 & -0.0003 & -0.0012 & -0.0010 & -0.0013 & 0.0007 & -0.0002 & -0.0002 & -0.0024 & -0.0003 \\
-0.0001 & -0.0002 & -0.0009 & -0.0004 & 0.0014 & 0.0016 & 0.0000 & -0.0006 & -0.0005 & -0.0008 & 0.0011 & -0.0001 & -0.0002 & -0.0017 & 0.0003 \\
-0.0001 & 0.0000 & -0.0001 & 0.0011 & -0.0003 & -0.0000 & 0.0015 & 0.0017 & 0.0015 & 0.0009 & -0.0005 & 0.0007 & -0.0000 & -0.0002 & -0.0008 \\
-0.0001 & 0.0001 & 0.0000 & 0.0050 & -0.0012 & -0.0006 & 0.0017 & 0.0168 & 0.0082 & 0.0079 & -0.0023 & -0.0036 & 0.0001 & 0.0005 & -0.0078 \\
-0.0003 & -0.0002 & -0.0008 & 0.0029 & -0.0010 & -0.0005 & 0.0015 & 0.0082 & 0.0129 & 0.0116 & -0.0021 & -0.0025 & 0.0007 & -0.0012 & -0.0072 \\
-0.0005 & -0.0004 & -0.0012 & 0.0032 & -0.0013 & -0.0008 & 0.0009 & 0.0079 & 0.0116 & 0.0200 & -0.0027 & -0.0034 & 0.0009 & -0.0011 & -0.0088 \\
-0.0003 & -0.0004 & -0.0017 & -0.0011 & 0.0007 & 0.0011 & -0.0005 & -0.0023 & -0.0021 & -0.0027 & 0.0140 & 0.0001 & 0.0004 & -0.0038 & 0.0168 \\
-0.0002 & 0 & 0 & 0 & -0.0002 & -0.0001 & 0.0007 & -0.0036 & -0.0025 & -0.0034 & 0.0001 & 0.0054 & -0.0001 & -0.0004 & 0.0028 \\
0.0004 & 0.0004 & -0.0026 & 0.0001 & -0.0002 & -0.0002 & -0.0000 & 0.0001 & 0.0007 & 0.0009 & 0.0004 & -0.0001 & 0.0103 & -0.0101 & 0.0028 \\
0.0003 & 0.0010 & 0.0111 & 0.0001 & -0.0024 & -0.0017 & -0.0002 & 0.0005 & -0.0012 & -0.0011 & -0.0038 & -0.0004 & -0.0101 & 0.0578 & -0.0094 \\
-0.0001 & -0.0002 & -0.0028 & -0.0026 & -0.0003 & 0.0003 & -0.0008 & -0.0078 & -0.0072 & -0.0088 & 0.0168 & 0.0028 & 0.0028 & -0.0094 & 0.1283
\end{bmatrix}$$

Gen	a	b	c	$P_{\min}$ [MW]	$P_{\max}$ [MW]
1	170.44	8.336	0.03073	40	80
2	309.54	70.706	0.02028	60	120
3	369.03	81.817	0.00942	80	190
4	135.48	69.467	0.08482	24	42
5	135.19	65.595	0.09693	26	42
6	222.33	80.543	0.01142	68	140
7	287.71	80.323	0.00357	110	300
8	391.98	6.999	0.00492	135	300
9	455.76	6.602	0.00573	135	300
10	722.82	12.908	0.00605	130	300
11	635.2	12.986	0.00515	94	375
12	654.69	12.796	0.00569	94	375
13	913.4	12.501	0.00421	125	500
14	1760.4	88.412	0.00752	125	500
15	1728.3	91.575	0.00708	125	500
16	1728.3	91.575	0.00708	125	500
17	1728.3	91.575	0.00708	125	500
18	647.85	79.691	0.00313	220	500
19	649.69	7.955	0.00313	220	500
20	647.83	79.691	0.00313	242	550
21	647.83	79.691	0.00313	242	550
22	785.96	66.313	0.00298	254	550
23	785.96	66.313	0.00298	254	550
24	794.53	66.611	0.00284	254	550
25	794.53	66.611	0.00284	254	550
26	801.32	71.032	0.00277	254	550
27	801.32	71.032	0.00277	254	550
28	1055.1	33.353	0.52124	10	150
29	1055.1	33.353	0.52124	10	150
30	1055.1	33.353	0.52124	10	150
31	1207.8	13.052	0.25098	20	70
32	810.79	21.887	0.16766	20	70
33	1247.7	10.244	0.2635	20	70
34	1219.2	83.707	0.30575	20	70
35	641.43	26.258	0.18362	18	60
36	1112.8	96.956	0.32563	18	60
37	1044.4	71.633	0.33722	20	60
38	832.24	16.339	0.23915	25	60
39	832.24	16.339	0.23915	25	60
40	1035.2	16.339	0.23915	25	60

Table 15. Data from the 40-unit generation system.

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